Review of Water-Entry Theory and Data

ALBERT MAY*
U.S. Naval Ordnance Laboratory, White Oak, Silver Spring, Md.

Water-entry problem areas include the impact force system, cavity growth and decay, and missile trajectory. Instrumentation suitable for measurement of impact deceleration has been developed, and is in use in current research. This includes studies for the development of scaling laws applicable to entries for which weight, buoyancy, and friction drag must be considered. Whip research is scant and the scaling of whip is not yet possible. Some research has been done on cavity shape and history, and on the pressure within the water-entry cavity. Design criteria for satisfactory water-entry stability and trajectory have seen little improvement.

Introduction

THE purpose of this review is to point out what is known about the water-entry process, the gaps in our knowledge, and the prospect of filling these gaps. It is not proposed to present the data which have been published.

In spite of the importance of water-entry information to antisubmarine warfare (ASW), precise data on impact deceleration are generally not available. The water-entry process is complicated and not easily adaptable to mathematical analysis. To make it more amenable to discussion, we shall assume that the missile enters the water with zero angle of attack whether the entry is vertical or oblique.

Water Impact

Forces

In the past both the prediction and measurement of the forces at water impact have proved most difficult. The major theoretical obstacle is the unknown change in the shape of the boundaries, a shape which must be determined as part of the solution. Many attacks have been made on the problem, but practical results have come almost entirely from approximations based on a method proposed over 40 years ago by von Kármán.[1]

Briefly, von Kármán assumed that momentum is conserved during the impact phase of water entry. The momentum of the water is that of the added mass, which is viewed as traveling with the missile at its instantaneous speed. The impact force arises from the acceleration of this water mass that was originally at rest. The method is at best a rough approximation, especially since the prediction of the added mass depends on the changing boundary, including the splash. Of the many predictions that have been based on the method, few appear realistic and these have been "corrected" by semiempirical approaches. The outstanding example of the effective use of such "corrections" is the analysis of Shiffman and Spencer for the vertical entry of spheres.[2]

Measurement of impact forces also has proved intractable because of the shock that the instruments must withstand and the rapid response required of them. Instrumentation is now catching up, and experimental measurements of deceleration are under way.

The measurement itself of the impact forces is not the whole problem by any means. Data have been published for the entry of missiles with a variety of nose shapes, of various sizes and weights, entering at various speeds; but the effects of mass, weight, buoyancy, and skin-friction drag have generally been completely disregarded. The result is that experimental data cannot be applied to a missile unless it is identical to that used in the experiment. Even if a missile differs only in weight, it is not evident how the experimental data can be applied. Shiffman and Spencer[2] laid an excellent foundation for a method of analysis. They assumed that for the water entry of a particular missile at a particular entry angle, the added mass is a function only of the distance the missile has penetrated the water. Of course, it is also proportional to the density of the water ρ and the cube of the missile radius r. The added mass is not a function, however, of the mass of the missile or of its speed.

This concept assumes that the water flow is related by simple geometry to the motion of the missile, and that the added mass has always the instantaneous speed of the missile. Although it is quite difficult to predict theoretically the value of this added mass, a method may be built on the premise that it has an effective value at each penetration distance, a value that can be determined experimentally from measurements of deceleration.

Complete details of the method cannot be given here, but the process is substantially the following, for an analysis based on measurements of deceleration as a function of depth. Assuming first that all forces other than the hydrodynamic forces on the nose can be neglected, the added mass M can be obtained from the conservation of momentum and can be written in the dimensionless form $M' = (2/\pi\rho r^3)M$. Figure 1 is a plot, from Ref. 2, of M' against b, the penetration distance in radii. It is for the vertical entry of spheres. It is easily shown that

$$dM'/db = C_{D\infty}^*$$

the instantaneous drag coefficient related to the impact speed, for a body of infinite mass. Figure 2, which is taken from the same source,[2] is a plot of impact drag coefficient against b, for various specific gravities of an entering sphere. The method that was derived for the prediction of the force on a sphere can be used for the correlation of experimental data from entries of spheres of various sizes and densities, and the extension of the method to other nose shapes and to oblique angles of entry is straightforward.

In analyzing data from the measurement of deceleration when nonhydrodynamic forces are acting, the hydrodynamic force is easily obtained by subtracting the effects of weight and buoyancy. When skin friction is significant, as in the case of the slender cone, it can be calculated also. Thus, we can compute the hydrodynamic force at any instant, but the determination of the added mass is more involved. The

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* Consultant, Aerospace Engineer.

simple momentum equation cannot be used, because weight, buoyancy, and skin friction prevent momentum from being conserved, and skin-friction drag even contributes to the added mass. Still another complication is the following. In the plots of Fig. 2, the graphs approach zero at large b, where the added mass is no longer changing. The actual force does not go to zero, however, but to a value determined by the steady cavity-running drag. Just how cavity-running drag builds up from zero at first contact, to the final value, is not known.

There are many problems here for which the solutions are not yet clear; but they are being worked on, and answers should be forthcoming.

Whip

Usually there is an impulsive change of angular velocity at water impact, and this has been given the name of "whip." No theoretical predictions of whip have been obtained, and measurements have been made principally at shallow angles of the order of 20°. Although for streamlined noses whip may be so large as to seriously affect the trajectory, our knowledge of it today is more qualitative than quantitative.

The treatment of whip is seriously hampered by our ignorance of its scaling laws. What is needed is a "whip coefficient" or "impulsive-moment coefficient" W which, for simple entry conditions, should have a dependence such as whip $= \Delta\omega = W\rho V d^4 \lambda / I$. Here the whip is assumed proportional to the water density and the impact speed V, to the fourth power of the diameter of the missile d, and inversely to the transverse moment of inertia I. In addition, whip is proportional to the (dimensionless) lever arm λ of the impulsive force.

The difficulties experienced in trying to use such a coefficient are overwhelming so far, and some of the difficulties involve the quantity λ. In the first place, since frictional forces are unimportant during impact, a right cylinder traveling axially will have forces parallel to the axis and the lever arm will be transverse; on the other hand, a hemisphere nose will have a center of pressure at the center of the sphere and the lever arm will be measured along the axis. More confusion arises from under-pressures which act along the lower surface of the nose during oblique entry. In this region the flow tends to separate from the missile surface but the air is unable for a time to maintain pressure within the separation layer. The underpressure force does not scale like the hydrodynamic force. Further, it changes with small variations in experimental conditions and modifies the scaling behavior. To date the scaling problem is completely unresolved. In general, the whip cannot be scaled even for changes of impact speed or the position of the center of gravity.

Accelerations within the missile

When impact forces are measured by means of models, the model is usually designed to be as rigid as possible so that the measured acceleration will be reasonably applicable to the whole missile. It must be recognized, however, that most prototype missiles are not of this simple rigid design. They are complicated structures in which the accelerations differ throughout the body. Often much higher decelerations can be measured toward the tail of the missile than just behind the nose. This is a problem to be studied by the structures engineer rather than the hydrodynamicist. Probably solutions must be sought separately for each structure.

Water-Entry Cavity

Cavity shape

The cavity that is generated by a missile at water entry controls the environment in which the entering missile travels. While cavity running, the tail fins of the missile exert no stabilizing influence except through collisions with the cavity wall, and the wall tends to guide the missile, usually on a curved trajectory; pressure-actuated devices on the missile are uninfluenced by the increasing hydrostatic head as the missile travels deeper, except as this pressure-increase influences the pressure in the cavity; batteries and other chambers, which are designed to flood, remain dry until the collapse of the cavity is nearly complete.

The persistence of the water-entry cavity—the length of time during which the cavity is attached to the missile, and the cavity's size during this period—has had little documentation. A number of reports have described the early history of the cavity, but the later cavity, long after it has closed at the water surface, has been neglected for two principal reasons: an experimental study of the late cavity requires a large testing facility in which photographic records can be obtained, and the late cavity is so variable as to require a statistical study of a large mass of experimental data. A recent report[3] makes some approach to this problem but the data do not extend to the time when the missile finally becomes cavity free.

Another recent report discusses the shape of the cavity while it is still attached to the water surface.[4] Still another[5] gives the formula $x/dC_D^{1/2} = 1.716(y/dC_D^{1/2})^2$ for the shape of the front portion of all cavities generated by missiles. According to the formula this front portion of the cavity is parabolic, as has long been stated; and all such cavity portions are geometrically similar, with the scale factor depending only on the drag area (C_DA) of the missile.

The application of this formula can be seen from the outlines in Fig. 3, which were obtained from vertical water entries with reduced air pressure above the water surface. The outlines were scaled with respect to the drag area, in accordance with the formula. If any of these outlines are superimposed, with the axes and the cross lines coinciding, they are found to be almost identical, except in the vicinity of the vertex. They were obtained from the following entries:

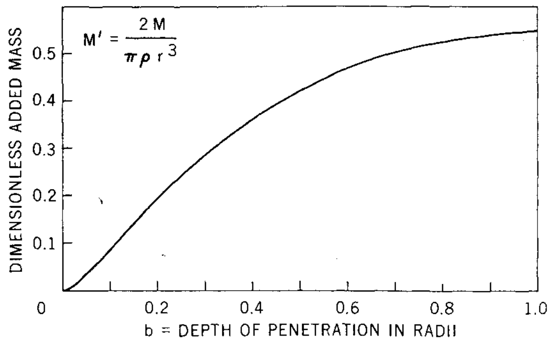

Fig. 1 Dimensionless virtual mass for a sphere entering water vertically (after Shiffman and Spencer).

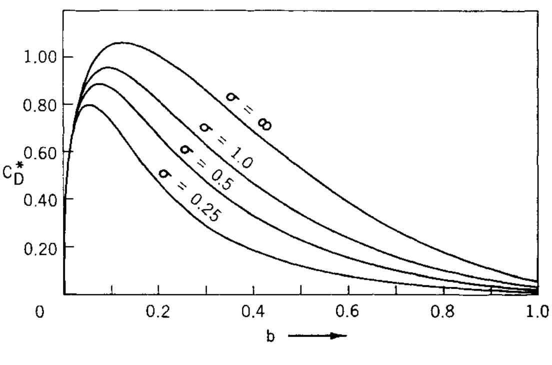

Fig. 2 Impact drag coefficient for spheres of various specific gravities (after Shiffman and Spencer).

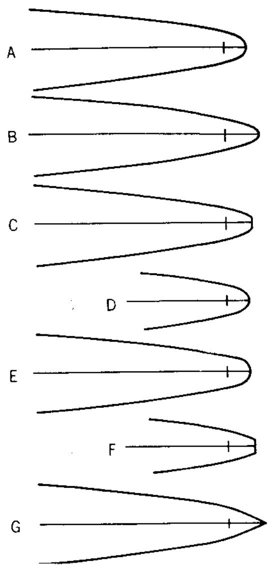

Fig. 3 Scaled cavities.

A) $1\frac{1}{2}$-in. sphere at 116 fps, B) plot from the formula given above, C) two-dimensional theoretical cavity with cavitation number of zero, D) $1\frac{1}{2}$-in. sphere at 41 fps, E) $\frac{1}{2}$-in. sphere at 208 fps, F) $\frac{1}{4}$-in. right cylinder at 37 fps, and G) 45° cone at 139 fps.

The portion of the cavity that can be so described will be small if large pressures act on the cavity, because it is assumed that this is the shape that would exist in the absence of all pressures except the hydrodynamic pressure directly caused by the cavity generation. At best this shape is an approximation, and the position of the cavity vertex is found only by trial. It is claimed that the formula gives the shape of the cavity of zero cavitation number and it is almost exactly similar to the theoretically predicted, two-dimensional cavity of zero cavitation number (sketch C).

Cavity pressure

Until recently only two isolated measurements of the pressure in the water-entry cavity had been reported, and theoretical predictions have been extremely primitive. Although the cavity is open to the atmosphere, it has usually been assumed that its pressure will be less than atmospheric by about $\frac{1}{2}\rho V^2$, a value derived from the assumption of steady flow into the cavity.

A program of cavity-pressure measurements has been made by H. I. Abelson at the Naval Ordnance Laboratory.[6] The details of this work will not be given here since it is in the process of publication. Some highlights of the research are the following.

Dr. Abelson's measurements were made principally for the vertical water entry of a cone with total angle of 140°, with a few measurements for a hemisphere, and a few for oblique entry. The models had a diameter of 3 in. and entered the water at speeds up to 250 fps. The pressure drop, below atmospheric, in the open cavity, was found to be an order of magnitude greater than $\frac{1}{2}\rho V^2$, with slightly lower pressures for the hemisphere than for the cone. As was expected, the maximum drop in cavity pressure was less for oblique than for vertical entry.

Trajectory After Water Entry

When a missile is cavity-running after water entry, it usually has a circular trajectory that is described while the tail of the missile rests against one cavity wall. Generally such a trajectory is strongly curved, and hence unsatisfactory. To obtain straight trajectories and to make missiles stay in the water when they enter at almost grazing angles, the only solution has been, and still is, to truncate the nose to provide a sufficient flat on the nose that the flow after separation from the nose does not strike the remainder of the missile. Unfortunately, the provision of a flat of sufficient size tends to give the missile large drag and large entry shock.

The trajectory can be straightened if the tail of the missile can be made to bounce from one cavity wall to the other. This can be achieved by putting a flared, or conical, skirt on the missile but usually the missile becomes too active in the cavity and the skirt causes large drag both while cavity running and especially when the missile becomes fully wetted. It would appear that a solution may be attainable by this technique if the skirt is given a minimum size, and perhaps is shed when the missile becomes fully wetted.

Conclusion

The mention of areas in which little or no progress has been made in solving the problems of water entry should not leave the impression that the situation is a hopeless one in which successful design engineering progress is impossible. Water-entry missiles can usually be designed to give a desired performance after water entry, but success still requires too much cut and try and too much sophisticated guessing for the situation to be entirely satisfactory. Much work is required before the water-entry regime can be treated on a wholly scientific and precise basis.

References

[1] Von Kármán, T., "The Impact of Seaplane Floats During Landing," TN 321, 1929, NACA.

[2] Shiffman, M. and Spencer, D. C., "The Force of Impact on a Sphere Striking a Water Surface (Second Approximation)," AMP Rept. 42.2R, July 1945, National Defense Research Committee.

[3] May, A., "A Study of the Water-Entry Cavity," TR 63-264, Dec. 1963, Naval Ordnance Lab., White Oak, Md.

[4] Abelson, H. I., "Cavity Shapes at Vertical Water Entry—A Comparison of Calculated and Observed Shapes," TR 67-31, Jan. 1967, Naval Ordnance Lab., White Oak, Md.

[5] May, A., "The Cavity After Vertical Water Entry," TR 68-114, July 1968, Naval Ordnance Laboratory, White Oak, Md.

[6] Abelson, H. I., "The Behavior of the Cavity Formed by a Projectile Entering the Water Vertically," Ph.D. thesis, June 1969, Univ. of Maryland, College Park, Md.